Geomatics Engineering Department

Second Year Geomatics

Geodesy 2 (GED209)

Lecture No: 2

Intersection and Resection

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Overview Of Previous Lecture

- Course Info.
- Scope
- Course Content
- Expected Learning Outcomes
- Course Assessment
- Teaching Members
- Lecture 1 – LOS Engineering
- Definition & Rationale
- Intervisibility Between Triangulation Stations
- Numerical Exercises
- Applications of LOS Engineering
- Software
- Summary

Overview Of Today's Lecture

- Basic Concepts
- Intersection
- Resection
- Three-Point Resection Problem
- Numerical Exercise
- Situations To Use Each Method
- Summary

Expected Learning Outcomes

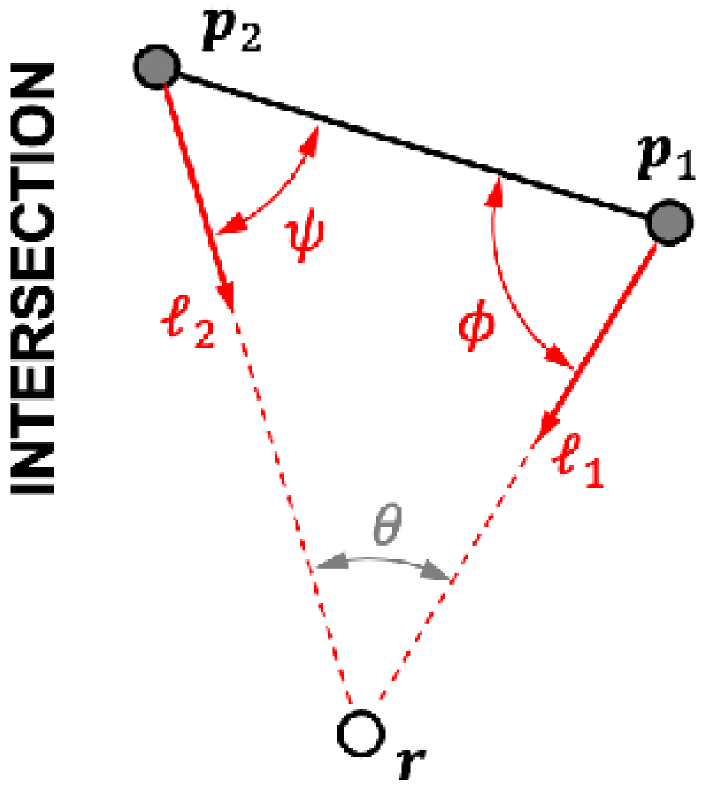
- Comprehend the concept of intersection in the context of surveying or geodesy.
- Understand the principles of resection and how it can be used in surveying or geodesy.
- Learn the mathematical equations and methods used to solve the three-point problem and apply them to real-world situations.
- Develop an understanding of when to use each method based on the characteristics of the problem or the available data.

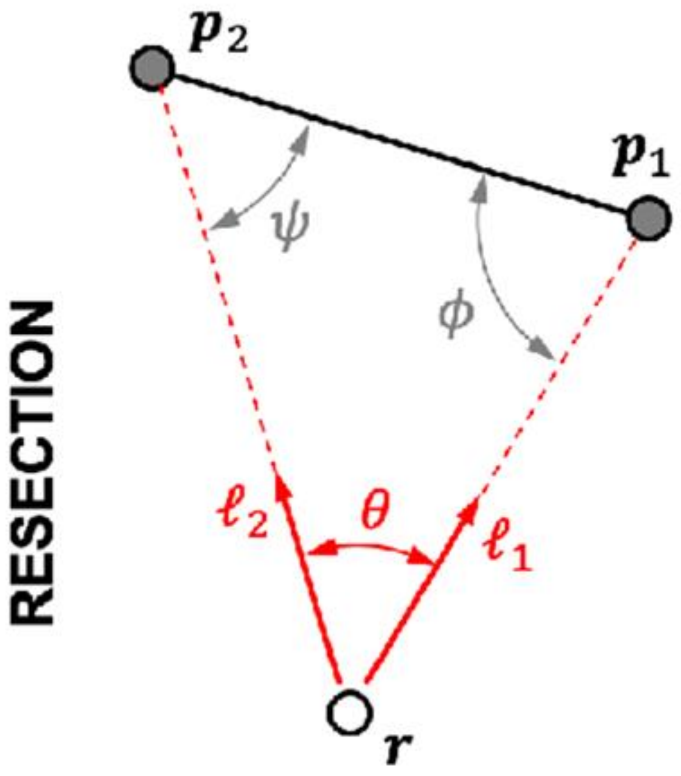
Basic Concepts

- ***Intersection***: the two points with known coordinates are occupied and sightings are taken to the unknown point.
- ***Resection***: the one point with unknown coordinates is occupied and sightings are taken to the known points.
- Using these techniques, one can establish the coordinates of a point P, by observations to or from known points. These techniques are useful for obtaining the position of single points, to provide control for setting out or detail survey.
- Measurements can be made with a compass, theodolite or with a total station using known points of a geodetic network or landmarks of a map.

Basic Concepts

Measurements originate from known vertices $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$.





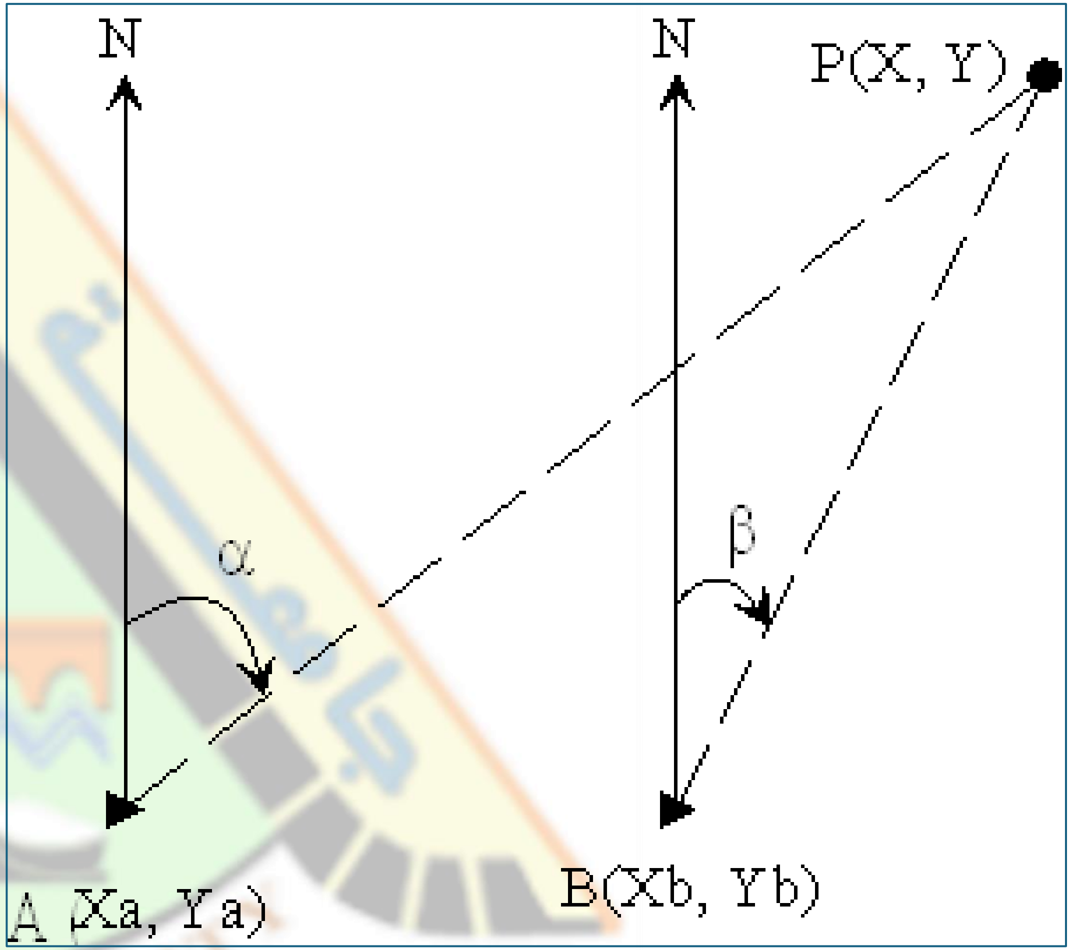
Measurements originate from unknown vertex $\boldsymbol{r}$.

* Images from internet

(1) Intersection

Intersection Problem

- Let the existing control stations are $A(X_A, Y_A)$ and $B(X_B, Y_B)$ and from which point $P(X, Y)$ is intersected.
- Also, bearing of AP is α, and bearing of BP is β.
- It is *assumed* that P is always to the right of A and B. in addition, α and β range from 0 to 90.



Intersection Problem

$$\tan\alpha = \frac{X - X_A}{Y - Y_A},$$

$$X - X_A = Y\tan\alpha - Y_A\tan\alpha$$

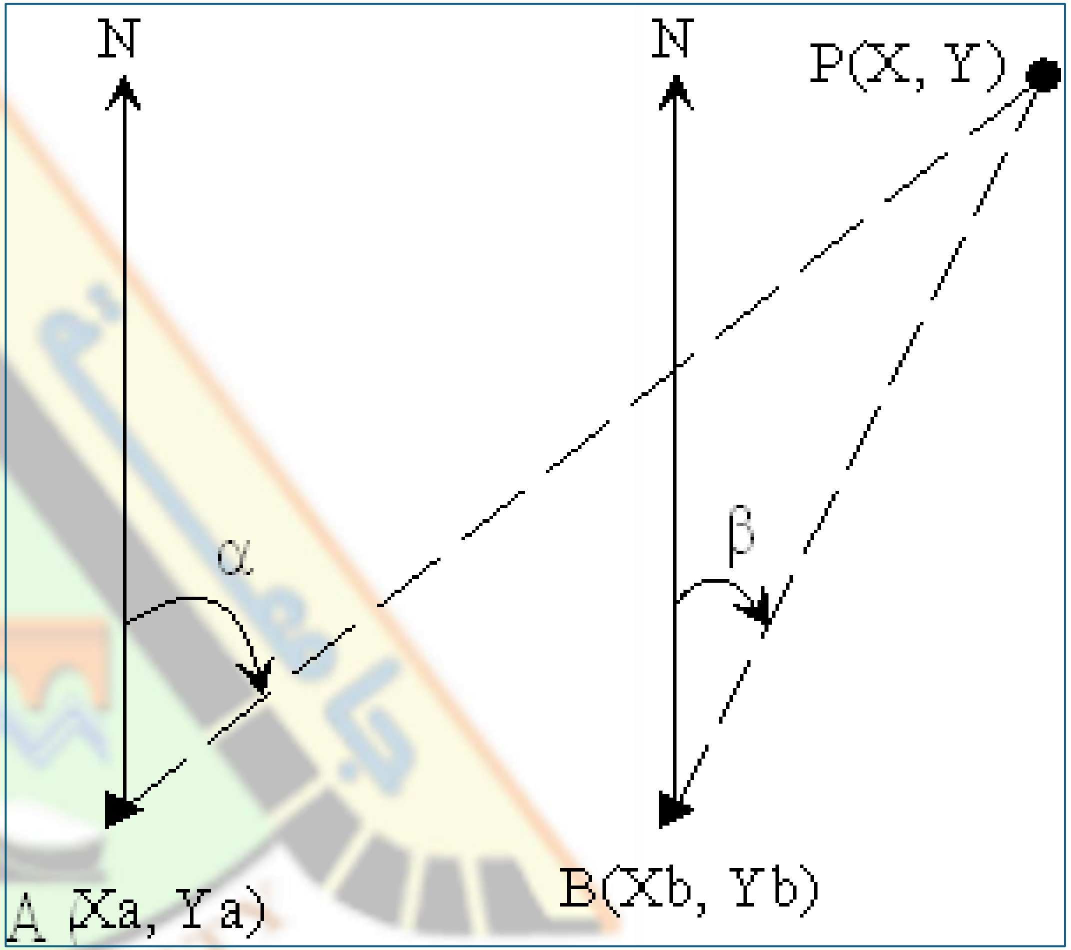
$$\therefore X = X_A + Y\tan\alpha - Y_A\tan\alpha \quad \text{\_\_\_\_\_\_\_\_(1)}$$

Similarly:

$$\tan\beta = \frac{X - X_B}{Y - Y_B},$$

$$X - X_B = Y\tan\beta - Y_B\tan\beta$$

$$\therefore X = X_B + Y\tan\beta - Y_B\tan\beta \quad \text{\_\_\_\_\_\_\_\_(2)}$$



Intersection Problem

Eq 1 = Eq 2, we get: -

$$X_A + Y\tan\alpha - Y_A\tan\alpha = X_B + Y\tan\beta - Y_B\tan\beta$$

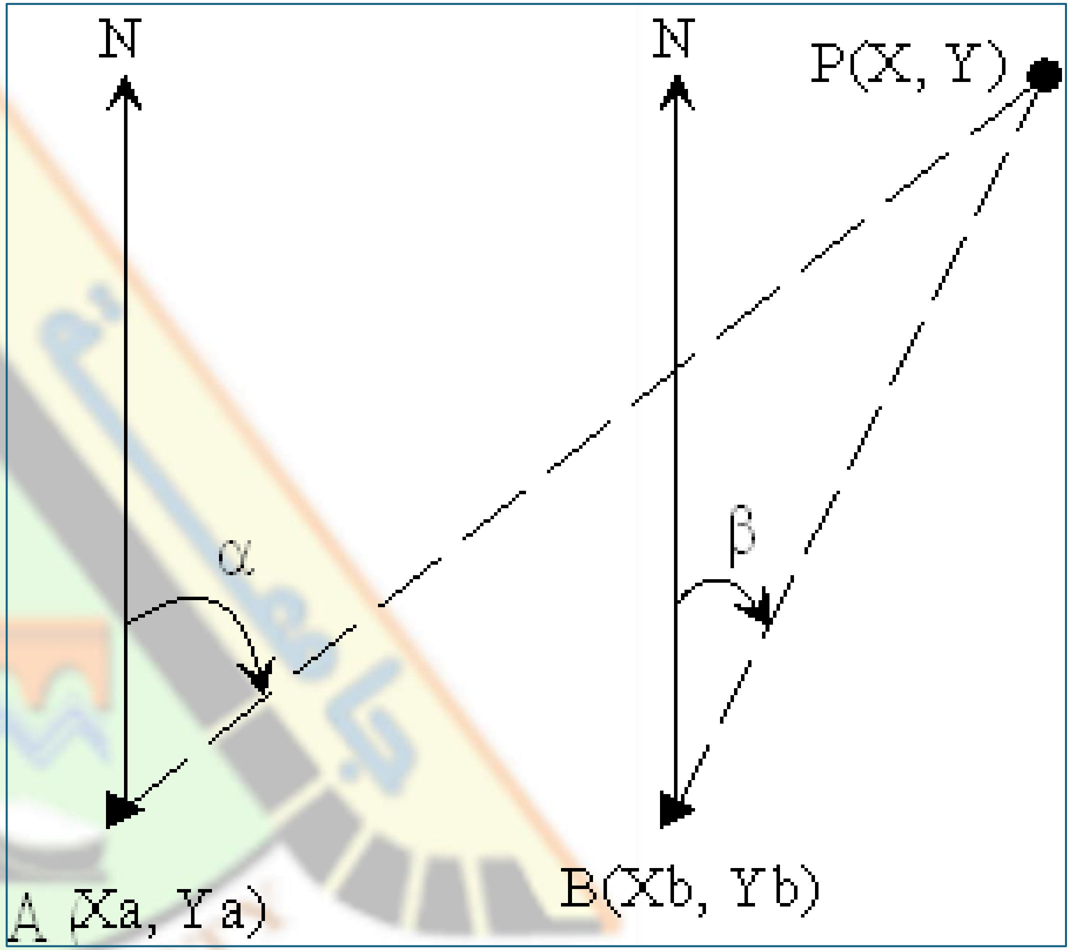
$$Y(\tan\alpha - \tan\beta) = X_B - X_A + Y\tan\alpha - Y_B\tan\beta$$

$$\therefore Y = \frac{X_B - X_A + Y_A\tan\alpha - Y_B\tan\beta}{(\tan\alpha - \tan\beta)} \quad (3)$$

Similarly:

$$\cot\alpha = \frac{Y - Y_A}{X - X_A}$$

$$Y = Y_A + X\cot\alpha - X_A\cot\alpha \quad (4)$$



Intersection Problem

$$\cot\beta = \frac{Y - Y_B}{X - X_B}$$

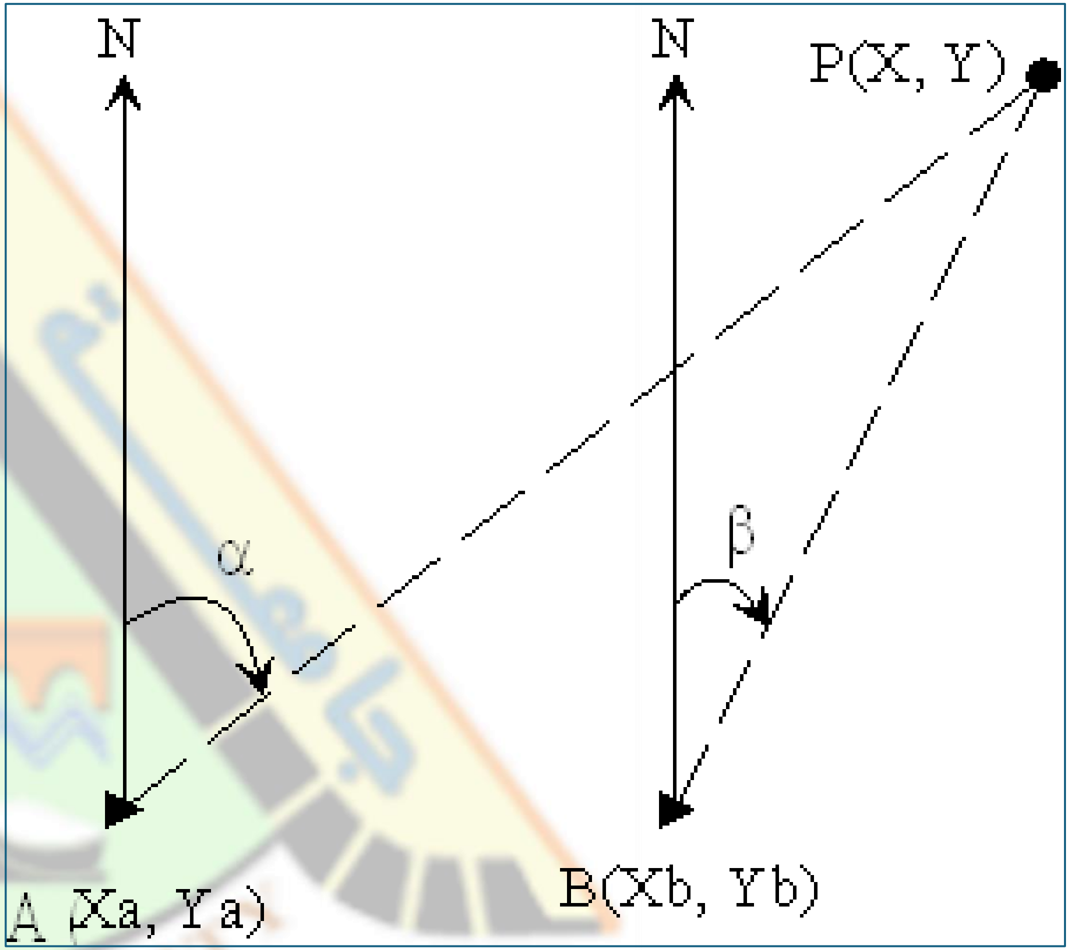
$$Y = Y_B + X\cot\beta - X_B\cot\beta \qquad (5)$$

Eq 4 = Eq. 5, we get:

$$Y_A + X\cot\alpha - X_A\cot\alpha = Y_B + X\cot\beta - X_B\cot\beta$$

$$X(\cot\alpha - \cot\beta) = Y_B - Y_A + X_A\cot\alpha - X_B\cot\beta$$

$$X = \frac{Y_B - Y_A + X_A\cot\alpha - X_B\cot\beta}{(\cot\alpha - \cot\beta)} \qquad (6)$$

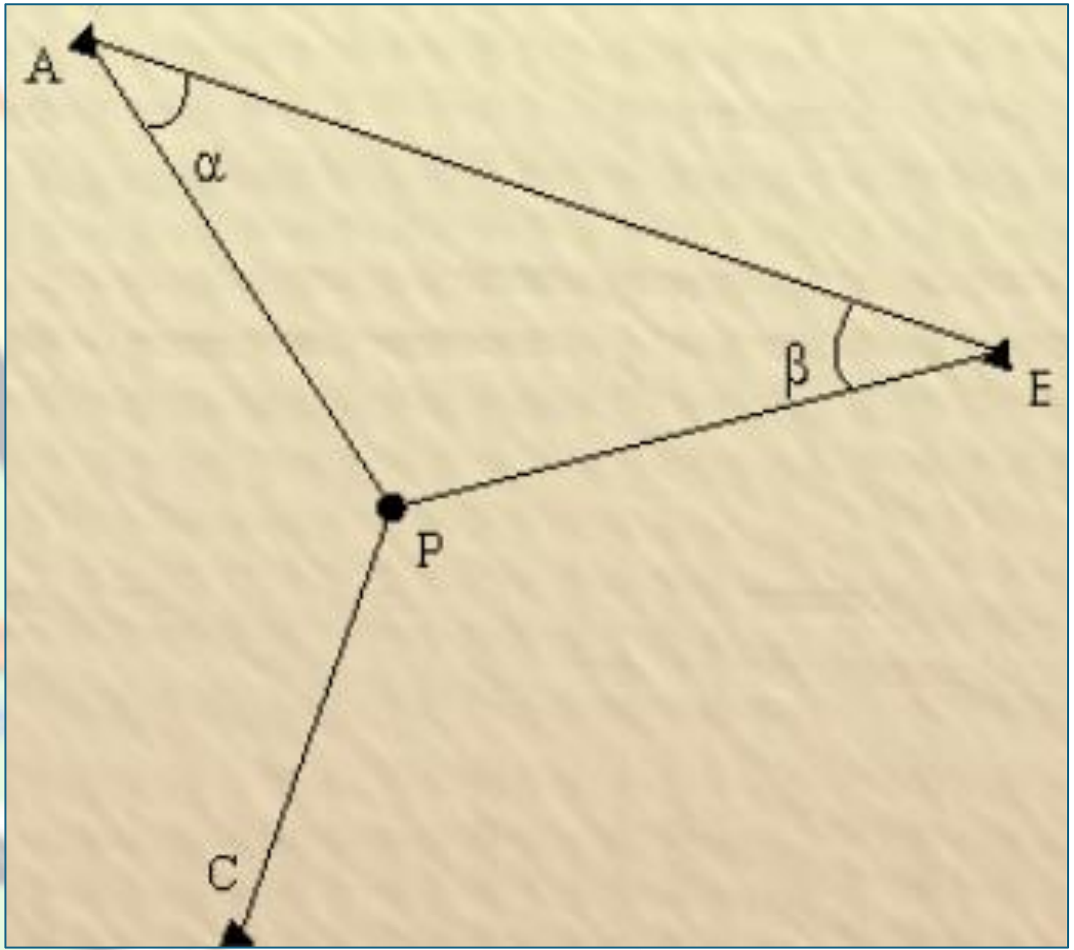


Intersection Problem

If the observed angles into P are used, the equations become: -

$$X = \frac{Y_B - Y_A + X_A \cot\beta + X_B \cot\alpha}{(\cot\alpha + \cot\beta)} \quad (7)$$

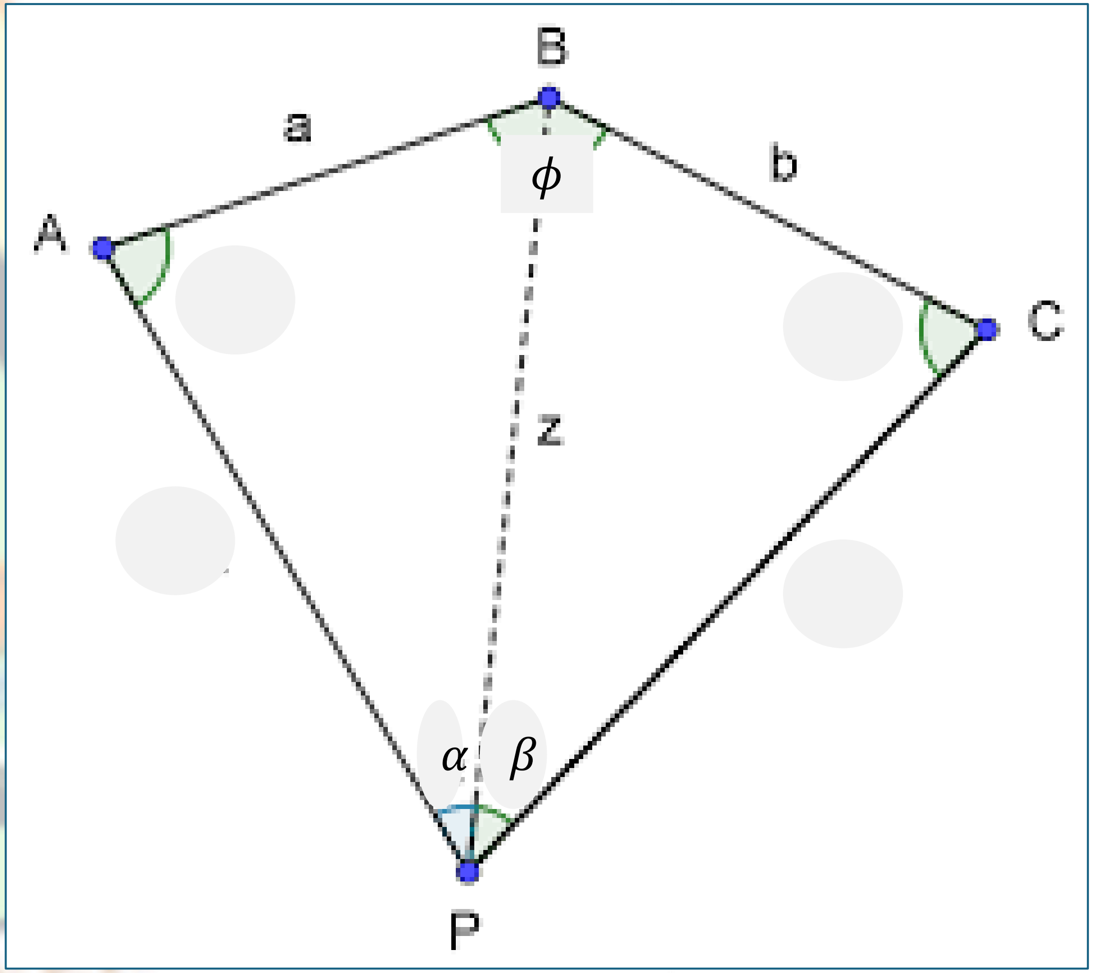
$$Y = \frac{X_A - X_B + Y_A \cot\beta + Y_B \cot\alpha}{(\cot\alpha + \cot\beta)} \quad (8)$$



(2) Resection (Three-point Problem)

THREE-POINT RESECTION PROBLEM

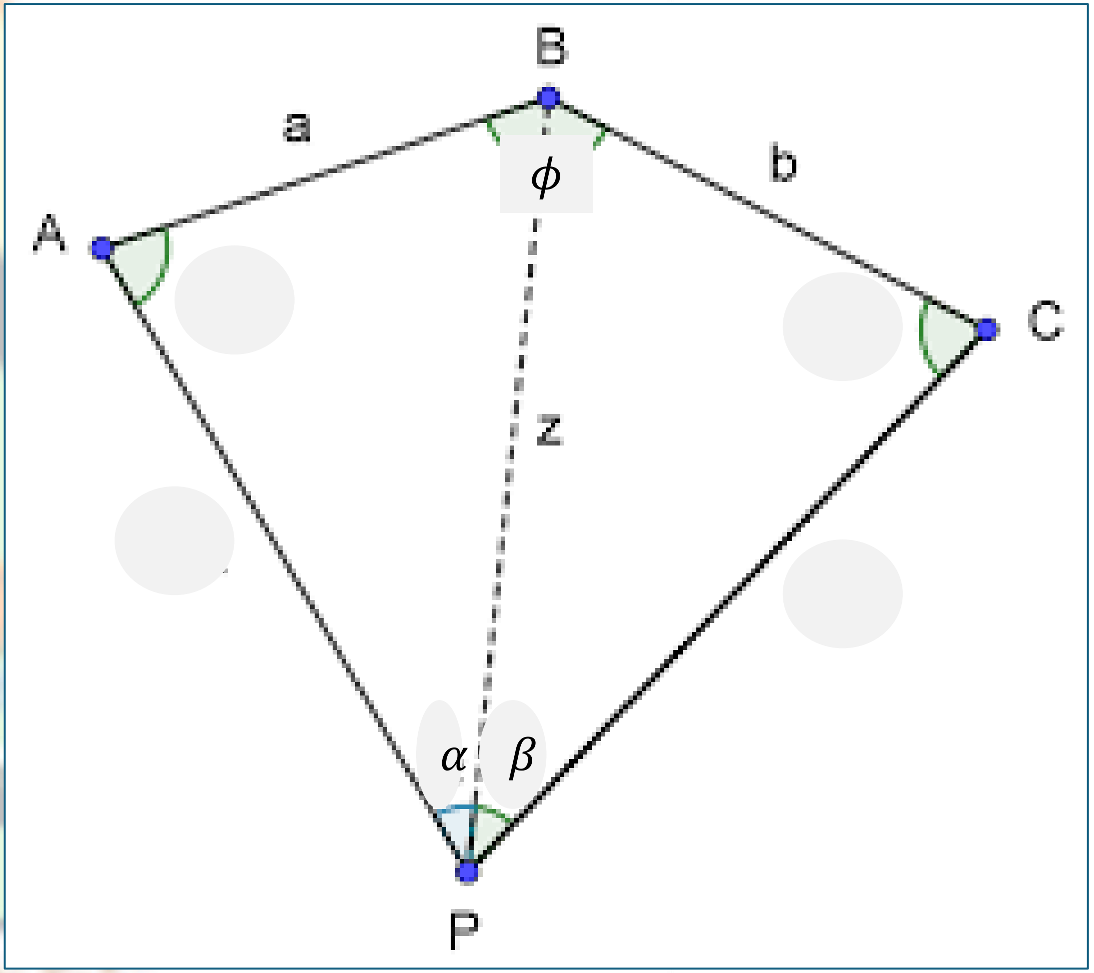
- Locates a single point by measuring horizontal angles from an unknown station to three visible control stations.
- It is considered a weaker solution than intersection.
- Useful technique for quickly fixing position where it is best required for setting out.
- Theodolite occupies station P, angles α and β are measured.



* Images from internet

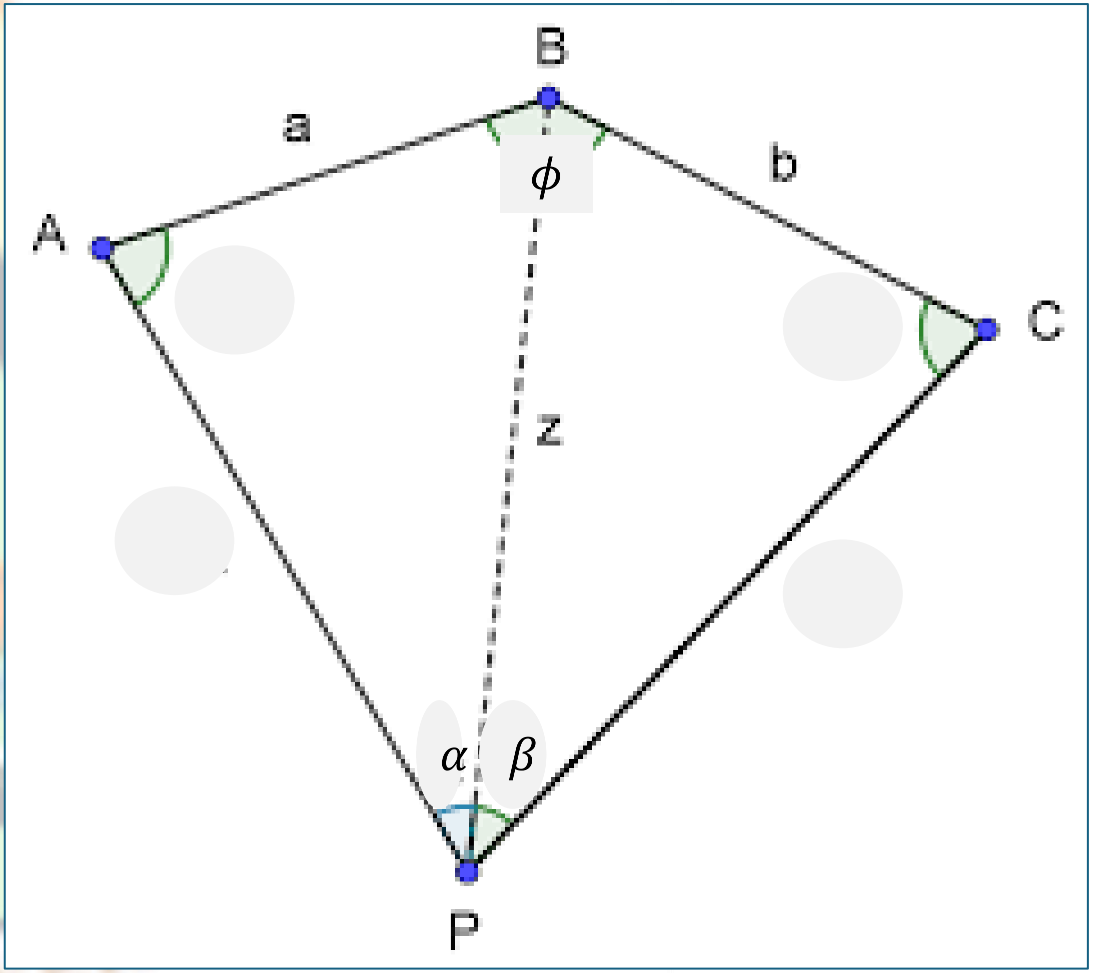
Three-Point Resection Problem

- Let $\angle BAP = \theta$, then $\angle BCP = (360 - \alpha - \beta - \phi) - \theta = S - \theta$
- ϕ is computed using the known coordinates of A, B, and C. Therefore, S is known.
- From $\Delta PAB, PB = \dfrac{BA \sin\theta}{\sin\alpha}$ ____________(9)
- From $\Delta PAB, PB = \dfrac{BC \sin(S - \theta)}{\sin\beta}$ __________(10)



THREE-POINT RESECTION PROBLEM

- Equating 9 and 10: -
- $\frac{BA \sin\theta}{\sin\alpha} = \frac{BC \sin(S-\theta)}{\sin\beta}$
- $\frac{\sin(S-\theta)}{\sin\theta} = \frac{BA \sin\beta}{BC \sin\alpha} = Q$
- $\frac{(\sin S \cos\theta - \cos S \sin\beta)}{\sin\theta} = Q$
- $\sin S \cot\theta - \cos S = Q$
- $\therefore \cot\theta = \frac{(Q+\cos S)}{\sin S}$ ______________ (11)
- Now, θ and $(S-\theta)$ are known, then the distances and bearings AP, BP, and CP are solved. Also, coordinates of P.



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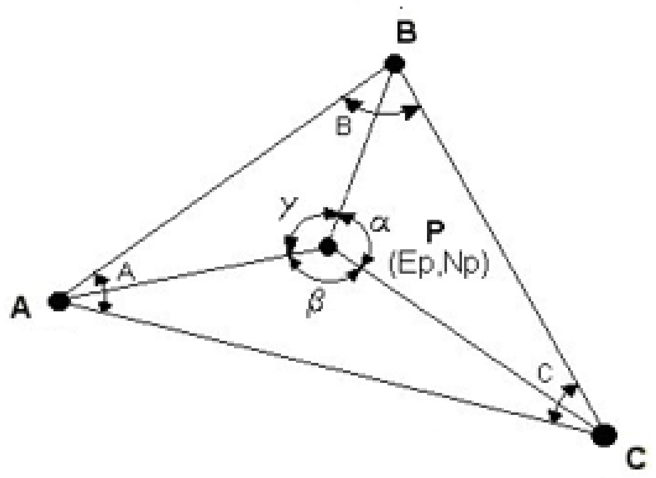
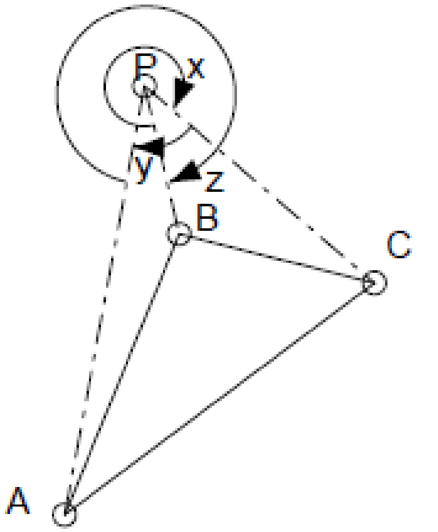
Notes on Three-Point Resection Problem

- This method fails if P lies on the circumference of a circle passing through A, B, and C. It means finite number of positions.
- In choosing resection station, care should be exercised such that it does not lie on the circumference of the **danger circle**.
- **The best position of P is: -**

1) Inside the ΔABC.
2) Well outside the circle which passes through A, B, and C.
3) Closer to the middle control station.





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Notes on Three-Point Resection Problem – Numerical Exercise

$\alpha = 41° \, 20' \, 35''$

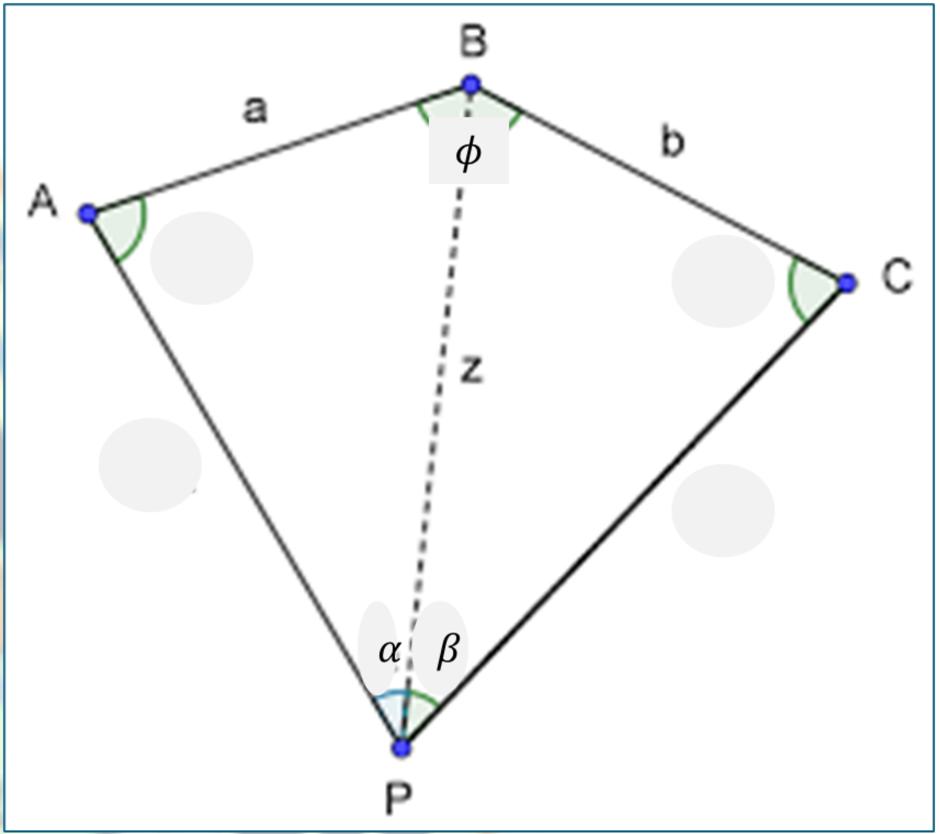
$\beta = 48° \, 53'$

$X_A = 5721.25, Y_A = 21802.48 \, m$

$X_B = 12963.71, Y_B = 27002.38 \, m$

$X_C = 20350.09, Y_C = 24861.22 \, m$

Compute the coordinates of P.



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NOTES ON THREE-POINT RESECTION PROBLEM – NUMERICAL EXERCISE

From the coordinates of A, B, and C: -

BC = 7690.460 m

AB = 8915.839 m

Bearing of AB = 54° 19′21.5“

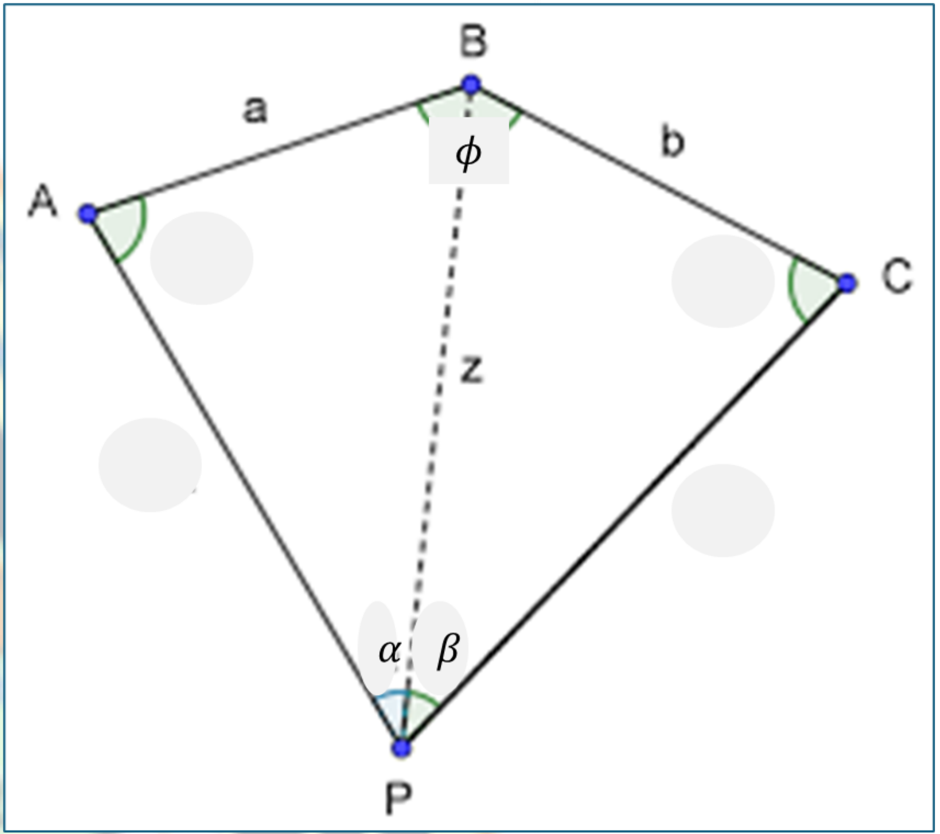
ϕ = 180 - 106° 09′56.8 – 54° 19′ 21.5 = 128° 09′24.6“

$S = (360 - \alpha - \beta - \phi)$ = 141° 36′48.4“

$$Q = \frac{AB \sin\beta}{BC \sin\alpha} = 1.322286$$

$$\cot\theta = \frac{(Q+\cos S)}{\sin S}$$

θ = 49° 04′15.5“



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Notes on Three-Point Resection Problem – Numerical Exercise

$$BP = \frac{AB \sin\theta}{\sin\alpha} = 10197.483\ m$$

$$BP = \frac{BC \sin(S-\theta)}{\sin\beta} = 10197.483\ m \text{ --- check}$$

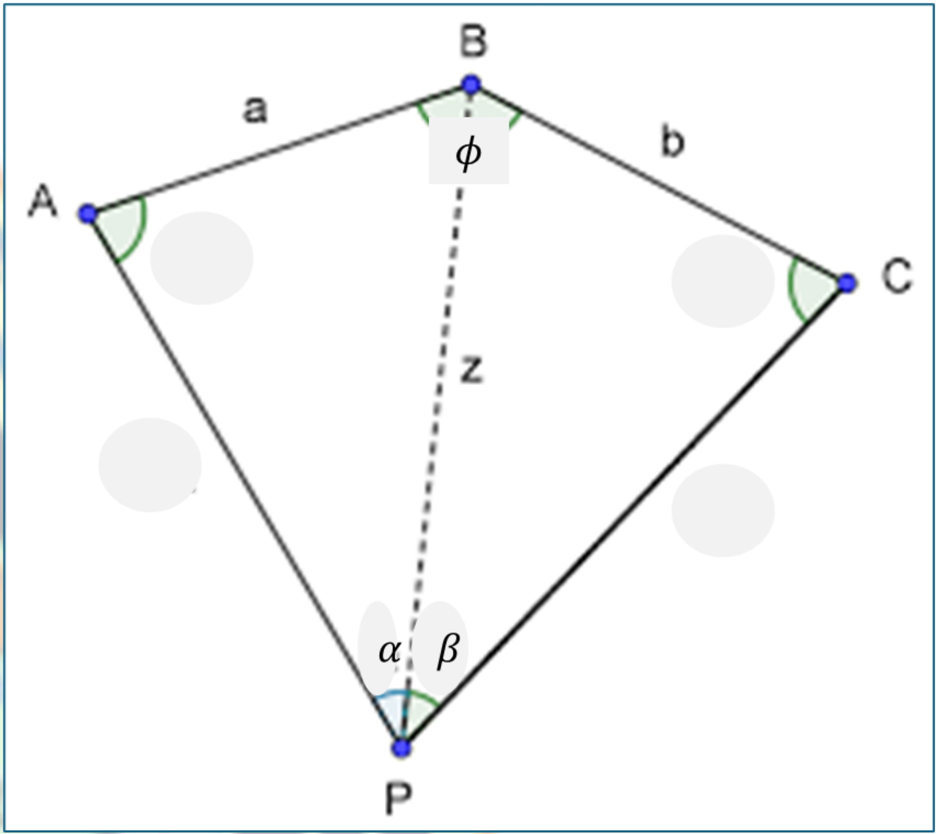
$$\angle CBP = 180 - [\beta + (s - \theta)] = 38.5708769°$$

Bearing of BP = $Bearing\ BC + \angle CBP = 144°\ 44'\ 12.0"$

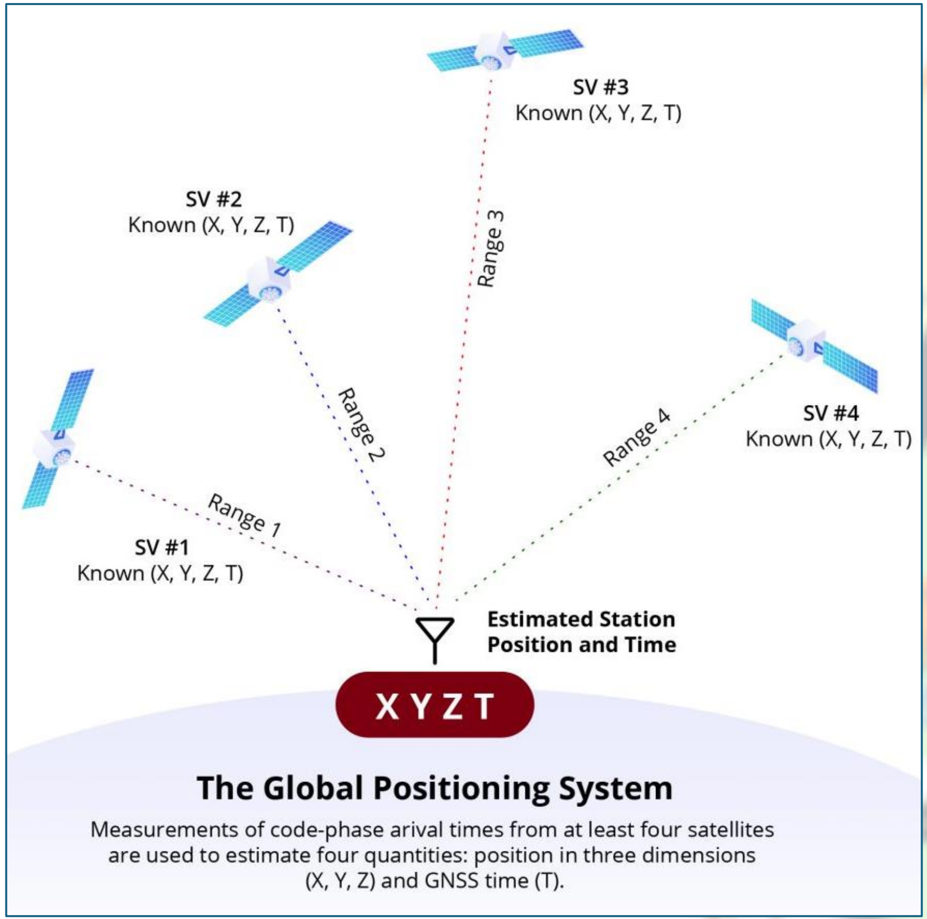
$$E_P = E_B + BP \sin(bearing\ BP) = 18851.076\ m$$

$$N_P = N_B + BP \cos(bearing\ BP) = 18676.061\ m$$

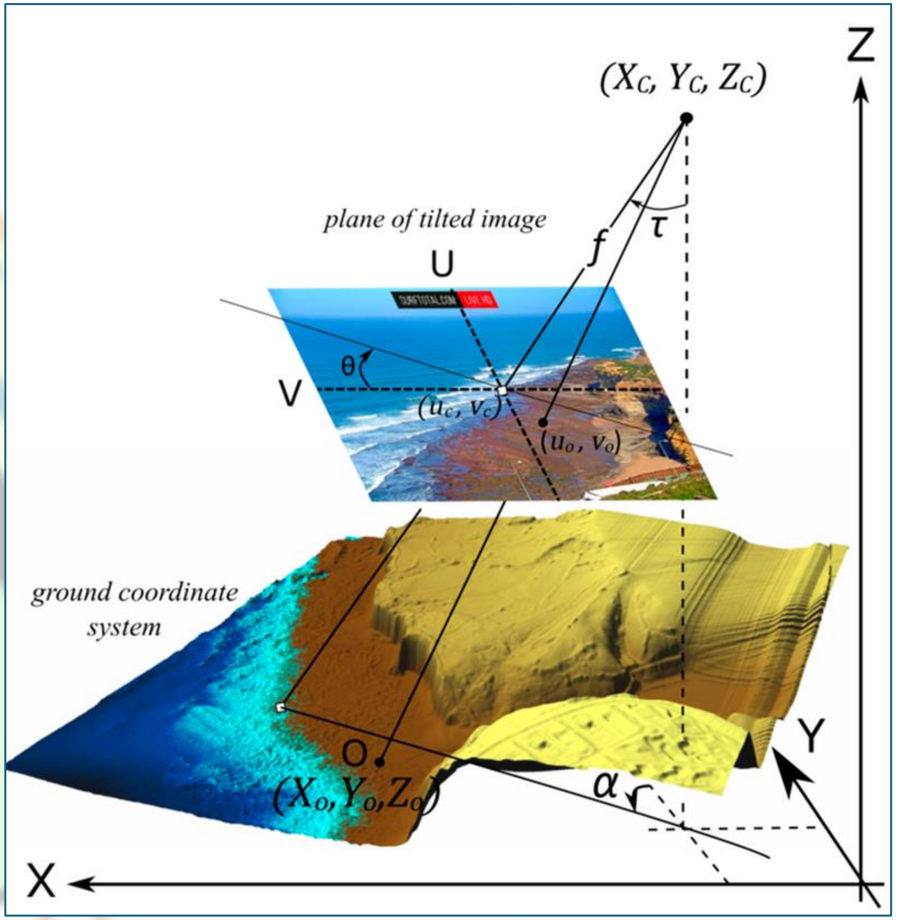
Checks can be made by computing the coordinates of P using the distance and bearing of AP and CP.



Common Systems Utilizing Resection



Satellite Positioning



Photogrammetry

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End Of Presentation

Thank You For Attention!